On Capacity Planning for Minimum Vulnerability

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Outline

- Introduction
- Network Criticality and Its Interpretations
- Vulnerability Analysis
- Optimizing Vulnerability Metrics
- Evaluations
- Conclusion

Introduction

- **Robustness** : to be insensitive to the environmental changes

- **Vulnerability**: deals with bottleneck points of the network
  - Perturb network parts or environmental parameters
  - Measure performance degradation

- So more robust networks are less vulnerable
  - Less sensitive to unwanted environmental changes

Objective of this work

- Network criticality (NC)
  - A global measure of robustness on a graph
  - Optimizing network criticality provides robustness

- Introduce vulnerability metrics
  - Based on the variations of NC in case of failures
- Optimize vulnerability metrics
  - to make the network less vulnerable to failures
- Compare the optimal weight sets
  - The metrics
  - Routing performance

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Network Criticality

- Based on Random Walks Betweenness
- Consider a weighted graph
- And a random walk from node s to d
- $b_{sk}(d)$ = average number of visits to node k
- Let $b_k = \sum_{s,d} b_{sk}(d)$
- It can be shown that b_k/W_k is independent of the node
- And it is a unique value which we name it ***Network Criticality***

Interpretations of NC

- **Average Resistance Distance**
- τ_{sd} =Resistance distance between nodes s and d
- τ_{sd} = effective resistance between two nodes when conductance's are equal to weights
- Network criticality is proportional to the average of τ_{sd}'s

$$\tau = \sum_i \sum_j \tau_{ij}$$

- **Average Sensitivity of betweenness to weight changes**

$$\tau = \frac{1}{m-1} \sum_{(i,j) \in E} \frac{\partial b_{ij}}{\partial w_{ij}}$$

Formulation of NC

- If $L = [l_{ij}]$ is the Laplacian of the graph,

  and $L^+ = [l_{ij}^+]$ is its Moore - Penrose inverse :

- $$\tau_{sd} = l_{ss}^+ + l_{dd}^+ - 2l_{sd}^+ \quad or \quad \tau_{sd} = u_{sd}^t L^+ u_{sd}$$

  $$\tau_{sd} = \frac{b_{sk}(d) + b_{dk}(s)}{W_k}$$

- $$\hat{\tau} = \frac{1}{n(n-1)} \sum_{s,d} \tau_{sd} = \frac{2}{n-1} Tr(L^+)$$

- $$\eta_k = \frac{b_k}{W_k} = \frac{n(n-1)}{2} \hat{\tau}$$

  $$\eta_{ij} = \frac{b_{ij}}{w_{ij}} = n(n-1)\hat{\tau}$$

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Vulnerability Analysis

- Study the behavior of NC when failures happen

- Definitions:
- $\hat{\tau}^{(ij)}$ = NC of the network when link (i, j) fails
- $\hat{\tau}^{(i)}$ = NC of the network when node i fails

- $max(\hat{\tau}^{(ij)})$ = the worst value NC takes in case of a single link failure
- The most critical link

- $max(\hat{\tau}^{(i)})$ = the worst value NC take in case of a single node failure
- The most critical Node
- we call these values ***Vulnerability Metrics***

Optimizing Vulnerability Metrics

- Optimizing the vulnerability metrics
- make the network less vulnerable to node or link failures

- Proposed Planning Optimizations:
- ***MMTL*** : ***M**inimize **M**aximum **T**au in case of **L**ink failures*
- ***MMTN*** : ***M**inimize **M**aximum **T**au in case of **N**ode failures*
- ***OT*** : ***O**ptimize **T**au*

- We assume:
- z_{ij} be the cost of assigning a unit of w_{ij} to link (i, j)
- Then the total planning cost for the network is:

$$\sum_{(i,j)\in E} z_{ij} w_{ij}$$

- And we consider a total budget of C for the network

MMTL Formulation

- ***MMTL*** Formulation:

$$Minimize \quad max(\hat{\tau}^{(ij)}) \quad \forall (i,j) \in E$$

$$Subject\ to \quad \sum_{(i,j)\in E} z_{ij} w_{ij} \leq C,\ C\ is\ fixed$$

$$w_{ij} \geq 0$$

- $\hat{\tau}$ is an strictly convex function of link weights
- $\hat{\tau}^{(ij)}$ is the NC of the network without link (i, j)
- So $\hat{\tau}^{(ij)}$ is convex as well
- And $max(\hat{\tau}^{(ij)})$ is convex since
- It is the maximum of some convex functions

- And this is a convex optimization problem
  - Convex objective
  - Linear constraints

MMTN and ***OT*** Formulation

- Again we can prove that $max(\hat{\tau}^{(i)})$ is a convex function of link weights and
- ***MMTN*** is a convex optimization problem as well:

$$\begin{aligned} &Minimize \quad max(\hat{\tau}^{(i)}) \ \ \forall i \in N \\ &Subject\ to \quad \sum_{(i,j)\in E} z_{ij} w_{ij} \leq C,\ C\ is\ fixed \\ &\qquad\qquad\quad w_{ij} \geq 0 \end{aligned}$$

- We use ***OT*** as a comparison base:

$$\begin{aligned} &Minimize \quad \hat{\tau} \\ &Subject\ to \quad \sum_{(i,j)\in E} z_{ij} w_{ij} \leq C,\ C\ is\ fixed \\ &\qquad\qquad\quad w_{ij} \geq 0 \end{aligned}$$

SDP Formulation

- SDP can be solved numerically faster for larger networks

$$\hat{\tau}^{(ij)} = \frac{2}{n-1} Tr(L^{(ij)+}) = \frac{2}{n-1} Tr(L^{(ij)} + J/n)^{-1} - \frac{2}{n-1}$$

- SDP formulation for ***MMTL***:

$$\begin{aligned} &Minimize && \frac{2}{n-1}\, t - \frac{2}{n-1} \\ &Subject\ to && Diag(Vec(W)).\overrightarrow{1} \leq C \\ & && \begin{pmatrix} \Gamma^{(ij)} & I \\ I & L^{(ij)} + \frac{J}{n} \end{pmatrix} \succeq 0 \qquad \forall (i,j) \in E \\ & && Tr(\Gamma^{(ij)}) \leq t \qquad \forall (i,j) \in E \\ & && Diag(Vec(W)) \succeq 0 \end{aligned}$$

- Schur complement of $\Theta_{ij} = \begin{pmatrix} \Gamma^{(ij)} & I \\ I & L^{(ij)} + \frac{J}{n} \end{pmatrix}$ is $\Gamma^{(ij)} - (L^{(ij)} + J/n)^{-1}$

- It is used to obtain this SDP formulation

Evaluations

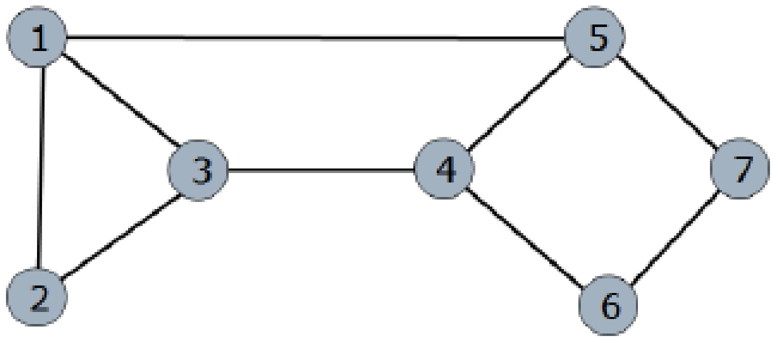
- Networks
  - Fish network
  - Rocketfuel Networks and Abilene
- CVX to solve SDPs
- 5 weight sets for each network

Method	Description	Optimized Metric
EW	Equal Weights	-
IW	Initial Weights	-
OT	Optimum $\hat{\tau}$	$\hat{\tau}$
MMTL	Minimize Maximum $\hat{\tau}$ in case of Link failures	$max(\hat{\tau}^{(ij)})$
MMTN	Minimize Maximum $\hat{\tau}$ in case of Node failures	$max(\hat{\tau}^{(i)})$

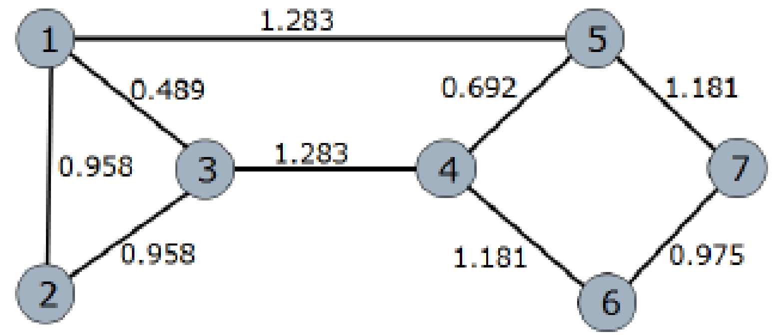
- Weights for EW:

$$w_{ij} = C / \left(\sum_{(i,j) \in E} z_{ij} \right) \quad \forall (i,j) \in E$$

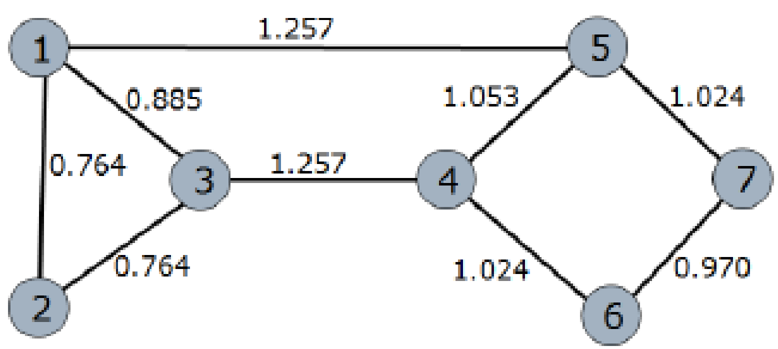
Weight sets for Fish Network



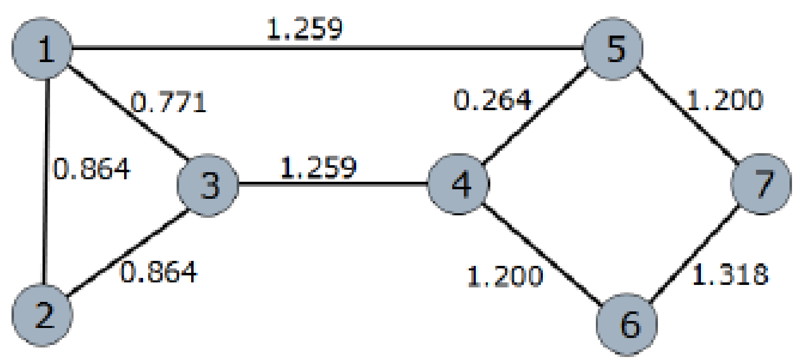
(a) EW



(b) OT



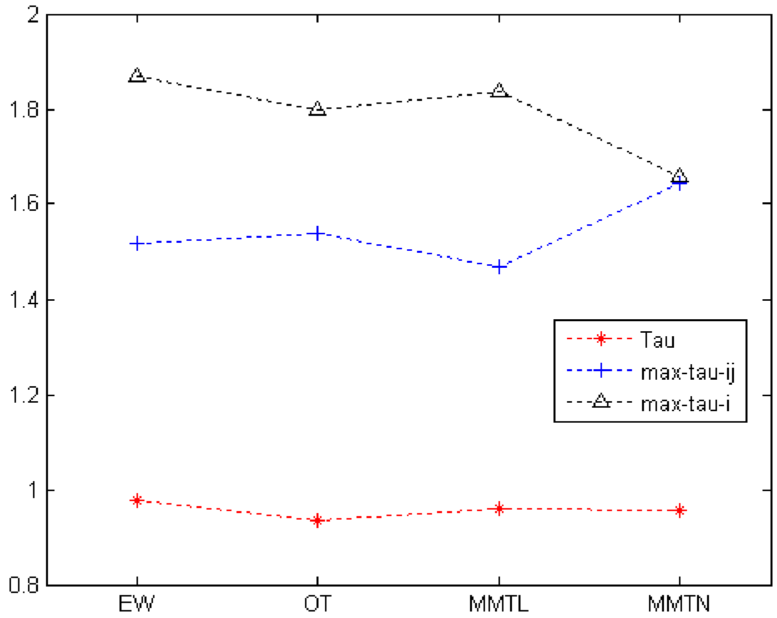
(c) MMTL



(d) MMTN

Fish Network Parameters

	$\hat{\tau}$	$max\ \hat{\tau}^{(ij)}$	$max\ \hat{\tau}^{(i)}$
EW	0.9756	1.5159	1.8667
OT	0.9334	1.5369	1.797
MMTL	0.9603	1.4679	1.8359
MMTN	0.9544	1.6435	1.6575



RocketFuel Dataset

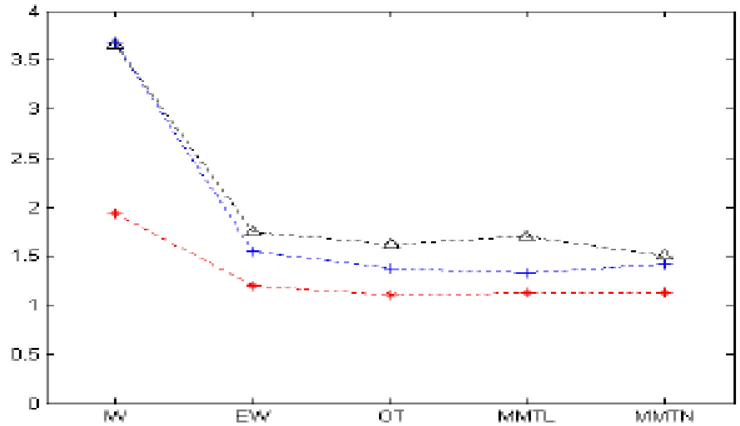
- Consolidate the nodes within a city
- Aggregate all the links between two cities
- Use OSPF weights to find out capacities
- OSPF weights are proportional to reciprocal of capacities

ISP	Routers	Links	Reduced Cities	Reduced Links	Weight per Link	Total Weight
1755	87	322	18	33	0.7822	51.628
3967	79	294	21	36	0.6743	48.551
1239	315	1944	30	69	0.7231	99.784

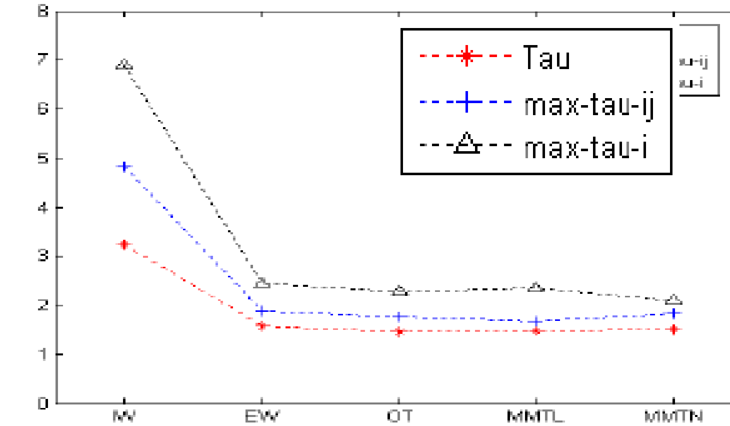
Parameters of 1755 Network

	$\hat{\tau}$	$max\ \hat{\tau}^{(ij)}$	$max\ \hat{\tau}^{(i)}$
IW	1.9408	3.6802	3.6557
EW	1.1977	1.5478	1.7525
OT	1.1013	1.3774	1.6188
MMTL	1.1239	1.3277	1.707
MMTN	1.1257	1.419	1.4997

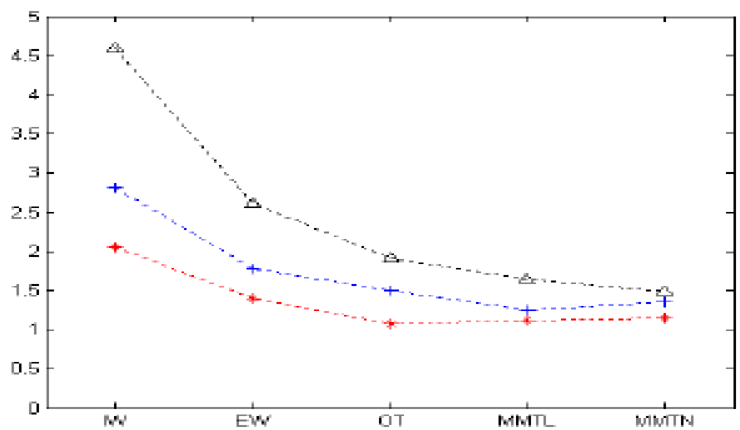
Comparison of IW and EW with Optimized Weight Sets



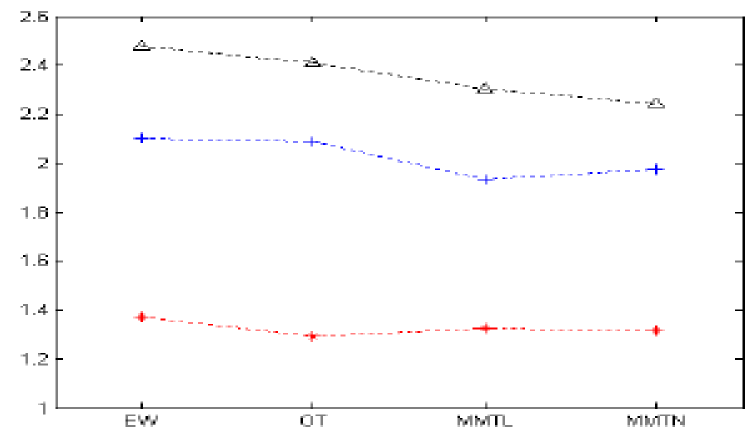
(a) 1755



(b) 3967

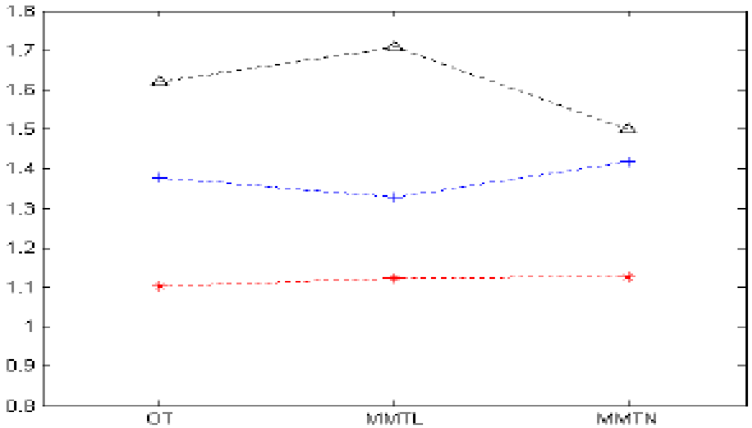


(c) 1239

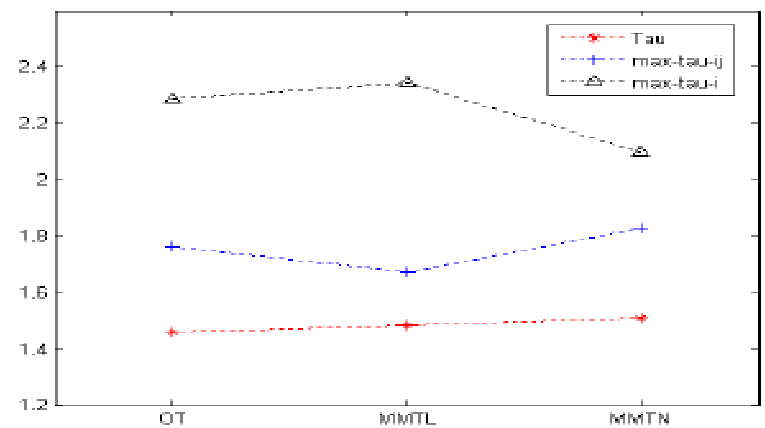


(d) Abilene

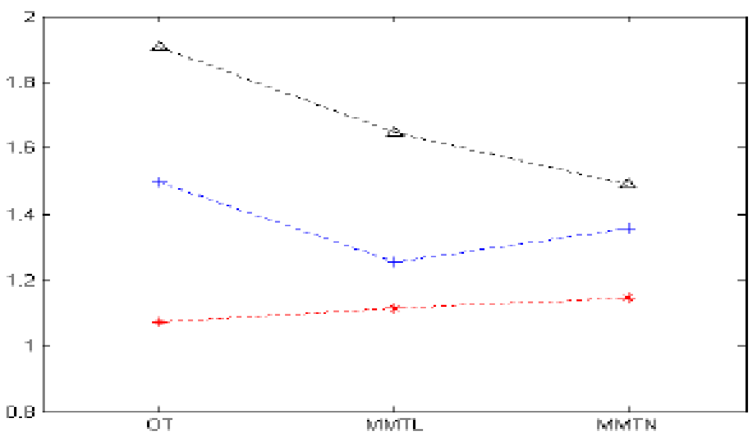
Comparison of Optimized Weight Sets



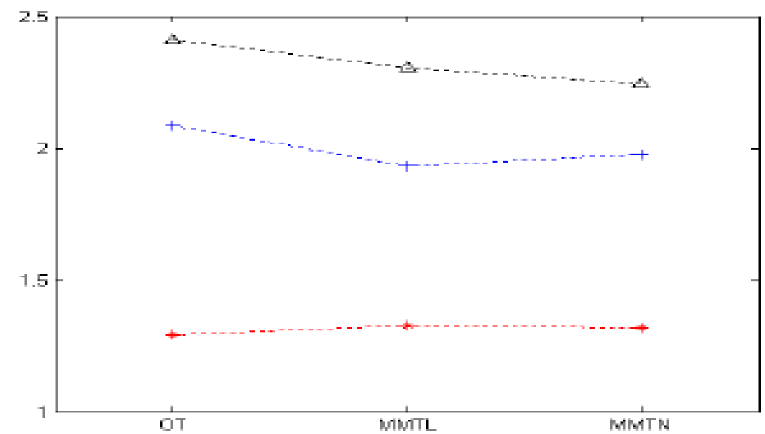
(a) 1755



(b) 3967



(c) 1239



(d) Abilene

Observations

- A huge gap between IW and optimized weight sets
- Our optimizations significantly improve the vulnerability of the networks

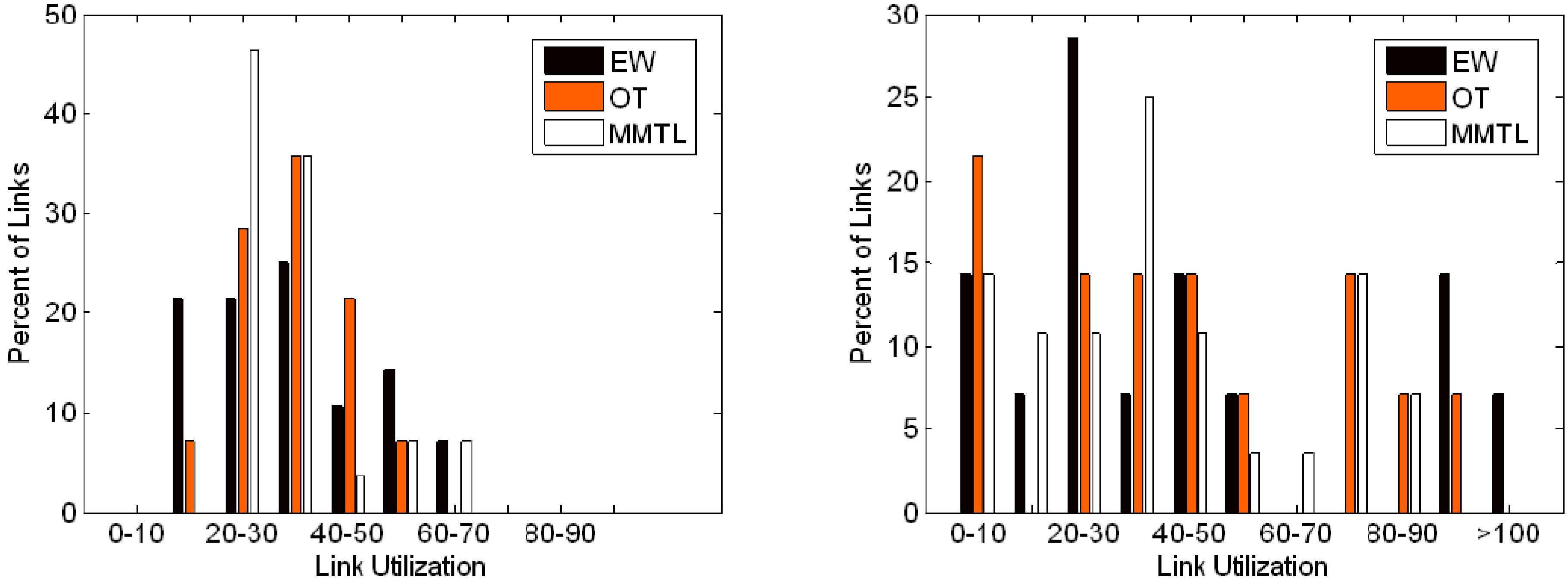
- Metric improvement for 1755 compared to IW
  - 42% for Tau
  - 61.4% for max-tau-ij
  - 60% for max-tau-i

- In different optimization methods
  - less variations in tau
  - More variations in max-tau-ij and max-tau-i

Routing Performance

- Applying the same traffic matrix
- Gravity Model to obtain traffic matrix
  - Input to each node proportional to capacity of links attached to it
  - It divides among the nodes proportional to capacity of links attached to them
- And the same routing method (independent)
- Using Totem Package
- IGP-WO weight optimizer to find OSPF optimal weights
- Uses Tabu search meta-heuristic method
- Capacity of links equal to the weight sets from our optimizations
- Examine link utilization before and after a failure

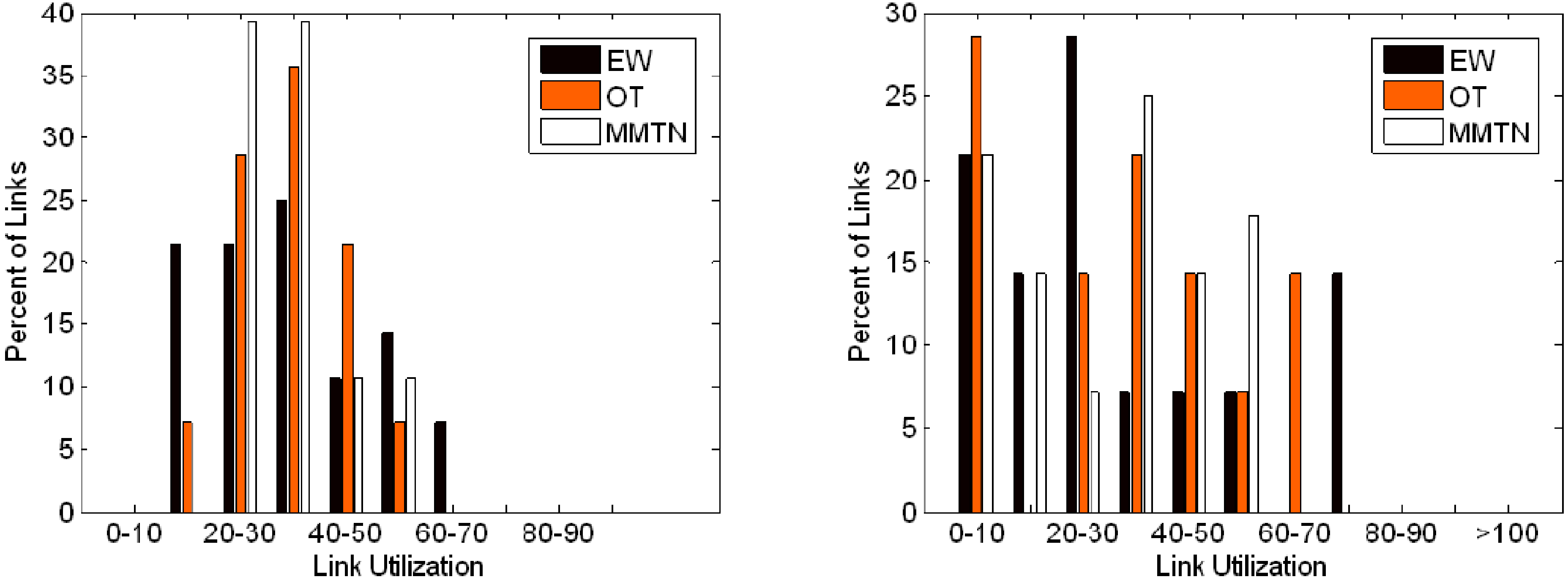
Link Utilization Before and After a Link Failure



(a) No Failure

(b) Single Link Failure

Link Utilization Before and After a Node Failure



(a) No Failure (b) Single Node Failure

Conclusion

- Define vulnerability metrics for a communication network
- Propose optimization problems to optimize these metrics
- Convert optimizations to SDP to be able to apply them on large networks

- Apply them on ISP topologies from Rocketfuel dataset
- Compare the metrics and routing performance using TOTEM package

- Future research
  - More than one failure
  - Correlated failures
  - Probabilistic approach to the failures
  - Compare to other graph metrics such as Algebraic Connectivity

Thank You