On Capacity Planning for Minimum Vulnerability

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Outline

- Introduction
- Network Criticality and Its Interpretations
- Vulnerability Analysis
- Optimizing Vulnerability Metrics
- Evaluations
- Conclusion
Introduction

- **Robustness**: to be insensitive to the environmental changes

- ** Vulnerability**: deals with bottleneck points of the network
  - Perturb network parts or environmental parameters
  - Measure performance degradation

- So more robust networks are less vulnerable
  - Less sensitive to unwanted environmental changes
Objective of this work

- Network criticality (NC)
  - A global measure of robustness on a graph
  - Optimizing network criticality provides robustness

- Introduce vulnerability metrics
  - Based on the variations of NC in case of failures

- Optimize vulnerability metrics
  - to make the network less vulnerable to failures

- Compare the optimal weight sets
  - The metrics
  - Routing performance
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Network Criticality

- Based on Random Walks Betweenness
- Consider a weighted graph
- And a random walk from node \( s \) to \( d \)
- \( b_{sk}(d) \) = average number of visits to node \( k \)

- Let \( b_k = \sum_{s,d} b_{sk}(d) \)
- It can be shown that \( b_k/W_k \) is independent of the node
- And it is a unique value which we name it **Network Criticality**
Interpretations of NC

- **Average Resistance Distance**
  \[ \tau_{sd} = \text{Resistance distance between nodes } s \text{ and } d \]
  \[ \tau_{sd} = \text{effective resistance between two nodes when conductance's are equal to weights} \]
  Network criticality is proportional to the average of \( \tau_{sd} \)'s

  \[ \tau = \sum_i \sum_j \tau_{ij} \]

- **Average Sensitivity of betweenness to weight changes**

  \[ \tau = \frac{1}{m-1} \sum_{(i,j) \in E} \frac{\partial b_{ij}}{\partial w_{ij}} \]
Formulation of NC

- If \( L = [l_{ij}] \) is the Laplacian of the graph,
  and \( L^+ = [l_{ij}^+] \) is its Moore-Penrose inverse:

- \( \tau_{sd} = l_{ss}^+ + l_{dd}^+ - 2l_{sd}^+ \text{ or } \tau_{sd} = u_{sd}^t L^+ u_{sd} \)

- \( \tau_{sd} = \frac{b_{sk}(d) + b_{dk}(s)}{W_k} \)

- \( \hat{\tau} = \frac{1}{n(n-1)} \sum_{s,d} \tau_{sd} = \frac{2}{n-1} Tr(L^+) \)

- \( \eta_k = \frac{b_k}{W_k} = \frac{n(n-1)}{2} \hat{\tau} \)

- \( \eta_{ij} = \frac{b_{ij}}{w_{ij}} = n(n-1) \hat{\tau} \)
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Vulnerability Analysis

- Study the behavior of NC when failures happen

- Definitions:
  - $\hat{\tau}^{(i,j)}$ = NC of the network when link $(i,j)$ fails
  - $\hat{\tau}^{(i)}$ = NC of the network when node $i$ fails

- $\text{max}(\hat{\tau}^{(i,j)})$ = the worst value NC takes in case of a single link failure
  - The most critical link

- $\text{max}(\hat{\tau}^{(i)})$ = the worst value NC take in case of a single node failure
  - The most critical Node
  - we call these values *Vulnerability Metrics*
Optimizing Vulnerability Metrics

- Optimizing the vulnerability metrics
- make the network less vulnerable to node or link failures

- Proposed Planning Optimizations:
  - **MMTL**: Minimize Maximum Tau in case of Link failures
  - **MMTN**: Minimize Maximum Tau in case of Node failures
  - **OT**: Optimize Tau

- We assume:
  - $z_{ij}$ be the cost of assigning a unit of $w_{ij}$ to link $(i, j)$
- Then the total planning cost for the network is:
  \[
  \sum_{(i, j) \in E} z_{ij} w_{ij}
  \]
- And we consider a total budget of $C$ for the network
**MMTL** Formulation:

- **MMTL** Formulation:
  
  \[
  \text{Minimize } \max(\hat{\tau}^{(ij)}) \quad \forall (i, j) \in E
  \]
  
  Subject to
  
  \[
  \sum_{(i, j) \in E} z_{ij} w_{ij} \leq C, \ C \text{ is fixed}
  \]
  
  \[
  w_{ij} \geq 0
  \]

- \(\hat{\tau}\) is an strictly convex function of link weights
- \(\hat{\tau}^{(ij)}\) is the NC of the network without link \((i, j)\)
- So \(\hat{\tau}^{(ij)}\) is convex as well
- And \(\max(\hat{\tau}^{(ij)})\) is convex since
- It is the maximum of some convex functions

- And this is a convex optimization problem
  - Convex objective
  - Linear constraints
**MMTN and OT Formulation**

- Again we can prove that $\max(\hat{r}^{(i)})$ is a convex function of link weights and

- **MMTN** is a convex optimization problem as well:

\[
\begin{align*}
\text{Minimize} & \quad \max(\hat{r}^{(i)}) \quad \forall i \in N \\
\text{Subject to} & \quad \sum_{(i,j) \in E} z_{ij}w_{ij} \leq C, \ C \text{ is fixed} \\
& \quad w_{ij} \geq 0
\end{align*}
\]

- We use **OT** as a comparison base:

\[
\begin{align*}
\text{Minimize} & \quad \hat{r} \\
\text{Subject to} & \quad \sum_{(i,j) \in E} z_{ij}w_{ij} \leq C, \ C \text{ is fixed} \\
& \quad w_{ij} \geq 0
\end{align*}
\]
**SDP Formulation**

- SDP can be solved numerically faster for larger networks:
  \[
  \hat{r}^{(ij)} = \frac{2}{n-1} Tr(L^{(ij)+}) = \frac{2}{n-1} Tr(L^{(ij)} + J/n)^{-1} - \frac{2}{n-1}
  \]

- SDP formulation for **MMTL**:
  
  \[
  \text{Minimize} \quad \frac{2}{n-1} t - \frac{2}{n-1}
  \]
  
  \[
  \text{Subject to} \quad \text{Diag}(\text{Vec}(W)). \mathbf{1}^T \leq C
  \]
  
  \[
  \left( \begin{array}{cc}
  \Gamma^{(ij)} & I \\
  I & L^{(ij)} + \frac{J}{n}
  \end{array} \right) \succeq 0 \quad \forall (i,j) \in E
  \]
  
  \[
  Tr(\Gamma^{(ij)}) \leq t \quad \forall (i,j) \in E
  \]
  
  \[
  \text{Diag}(\text{Vec}(W)) \succeq 0
  \]

- Schur complement of \( \Theta_{ij} = \left( \begin{array}{cc}
  \Gamma^{(ij)} & I \\
  I & L^{(ij)} + \frac{J}{n}
  \end{array} \right) \) is
  \[
  \Gamma^{(ij)} - (L^{(ij)} + J/n)^{-1}
  \]

- It is used to obtain this SDP formulation
Evaluations

- Networks
  - Fish network
  - Rocketfuel Networks and Abilene
- CVX to solve SDPs
- 5 weight sets for each network

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
<th>Optimized Metric</th>
</tr>
</thead>
<tbody>
<tr>
<td>EW</td>
<td>Equal Weights</td>
<td>-</td>
</tr>
<tr>
<td>IW</td>
<td>Initial Weights</td>
<td>-</td>
</tr>
<tr>
<td>OT</td>
<td>Optimum $\hat{\tau}$</td>
<td>$\hat{\tau}$</td>
</tr>
<tr>
<td>MMTL</td>
<td>Minimize Maximum $\hat{\tau}$ in case of Link failures</td>
<td>$\max(\hat{\tau}^{(ij)})$</td>
</tr>
<tr>
<td>MMTN</td>
<td>Minimize Maximum $\hat{\tau}$ in case of Node failures</td>
<td>$\max(\hat{\tau}^{(i)})$</td>
</tr>
</tbody>
</table>

- Weights for EW:

\[
\begin{align*}
    w_{ij} &= C / \left( \sum_{(i,j) \in E} z_{ij} \right) & \forall (i, j) \in E
\end{align*}
\]
Weight sets for Fish Network

(a) EW

(b) OT

(c) MMTL

(d) MMTN
Fish Network Parameters

<table>
<thead>
<tr>
<th></th>
<th>$\hat{\tau}$</th>
<th>$\text{max } \hat{\tau}^{(ij)}$</th>
<th>$\text{max } \hat{\tau}^{(i)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>EW</td>
<td>0.9756</td>
<td>1.5159</td>
<td>1.8667</td>
</tr>
<tr>
<td>OT</td>
<td>0.9334</td>
<td>1.5369</td>
<td>1.797</td>
</tr>
<tr>
<td>MMTL</td>
<td>0.9603</td>
<td>1.4679</td>
<td>1.8359</td>
</tr>
<tr>
<td>MMTN</td>
<td>0.9544</td>
<td>1.6435</td>
<td>1.6575</td>
</tr>
</tbody>
</table>
RocketFuel Dataset

- Consolidate the nodes within a city
- Aggregate all the links between two cities
- Use OSPF weights to find out capacities
- OSPF weights are proportional to reciprocal of capacities

<table>
<thead>
<tr>
<th>ISP</th>
<th>Routers</th>
<th>Links</th>
<th>Reduced Cities</th>
<th>Reduced Links</th>
<th>Weight per Link</th>
<th>Total Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>1755</td>
<td>87</td>
<td>322</td>
<td>18</td>
<td>33</td>
<td>0.7822</td>
<td>51.628</td>
</tr>
<tr>
<td>3967</td>
<td>79</td>
<td>294</td>
<td>21</td>
<td>36</td>
<td>0.6743</td>
<td>48.551</td>
</tr>
<tr>
<td>1239</td>
<td>315</td>
<td>1944</td>
<td>30</td>
<td>69</td>
<td>0.7231</td>
<td>99.784</td>
</tr>
</tbody>
</table>
## Parameters of 1755 Network

<table>
<thead>
<tr>
<th></th>
<th>$\hat{\tau}$</th>
<th>$max\ \hat{\tau}^{(ij)}$</th>
<th>$max\ \hat{\tau}^{(i)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>IW</td>
<td>1.9408</td>
<td>3.6802</td>
<td>3.6557</td>
</tr>
<tr>
<td>EW</td>
<td>1.1977</td>
<td>1.5478</td>
<td>1.7525</td>
</tr>
<tr>
<td>OT</td>
<td>1.1013</td>
<td>1.3774</td>
<td>1.6188</td>
</tr>
<tr>
<td>MMTL</td>
<td>1.1239</td>
<td>1.3277</td>
<td>1.707</td>
</tr>
<tr>
<td>MMTN</td>
<td>1.1257</td>
<td>1.419</td>
<td>1.4997</td>
</tr>
</tbody>
</table>
Comparison of IW and EW with Optimized Weight Sets

(a) 1755

(b) 3967

(c) 1239

(d) Abilene
Comparison of Optimized Weight Sets

(a) 1755

(b) 3967

(c) 1239

(d) Abilene
Observations

- A huge gap between IW and optimized weight sets
- Our optimizations significantly improve the vulnerability of the networks

- Metric improvement for 1755 compared to IW
  - 42% for Tau
  - 61.4% for max-tau-ij
  - 60% for max-tau-i

- In different optimization methods
  - less variations in tau
  - More variations in max-tau-ij and max-tau-i
Routing Performance

- Applying the same traffic matrix
- Gravity Model to obtain traffic matrix
  - Input to each node proportional to capacity of links attached to it
  - It divides among the nodes proportional to capacity of links attached to them

- And the same routing method (independent)
- Using Totem Package
- IGP-WO weight optimizer to find OSPF optimal weights
- Uses Tabu search meta-heuristic method

- Capacity of links equal to the weight sets from our optimizations

- Examine link utilization before and after a failure
Link Utilization Before and After a Link Failure

(a) No Failure

(b) Single Link Failure
Link Utilization Before and After a Node Failure

(a) No Failure

(b) Single Node Failure
Conclusion

- Define vulnerability metrics for a communication network
- Propose optimization problems to optimize these metrics
- Convert optimizations to SDP to be able to apply them on large networks
- Apply them on ISP topologies from Rocketfuel dataset
- Compare the metrics and routing performance using TOTEM package

Future research
- More than one failure
- Correlated failures
- Probabilistic approach to the failures
- Compare to other graph metrics such as Algebraic Connectivity
Thank You